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Normal and supernormal detonation velocity

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With 1 figure in the text

As a consequence of the hydrodynamic theory of detonation every velocity of a plane stationary detonation wave is a supernormal one if it differs from the normal one calculated according to the Chapman-Jouguet condition. Supernormal velocities may be obtained in two ways quite different in principle corresponding to different parts of the Hugoniot curve. This is well known but nevertheless I think there are still consequences which deserve to be pointed out. In the present paper we will study the distribution of pressure and flow velocity behind the reaction front. In principle this was already done by G. Taylor [1]. It is the same idea which has been applied in the more general case considered here.

We assume the charge to be cylindrical and enclosed in an absolutely undeformable tube of the same diameter. The one end face of the charge in which initiation takes place lies against a piston which moves with constant velocity $\gtreqless 0$ from the moment of initiation. The length of the reaction zone is assumed to be ignorable.

Signs.

t	the time, $t=0$ at the moment of initiation
x	distance from the plane of initiation
p	pressure
ϱ	density
D	detonation velocity
v	particle velocity
w	piston velocity
$c=\sqrt{\left(\frac{dp}{d\varrho}\right)_{\text{ad.}}}$	sound velocity
$\xi=\frac{x}{Dt}$	
n	fraction of explosive energy released.

Index 0 indicates the intact explosive, index 1 the detonation gas immediately behind the reaction zone and an asterisk (D^*, p_1^* etc.) that the detonation occurs according to the Chapman-Jouguet condition.

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At the time t the front is at the distance Dt from the initiation plane $x=0$. If we assume an adiabatic process behind the front the quantities p, ϱ etc. vill be functions of ξ and we consequently have

$$dp=\frac{dp}{d\xi}\left(\frac{dx}{Dt}-\frac{x\,dt}{Dt^2}\right).$$

Accordingly $dp=0$ if at least one of the conditions

$$\frac{dp}{d\xi}=0$$

$$\frac{dx}{dt}=\xi D$$

are fulfilled.

In a point ξ where $dp/d\xi \neq 0$ the state therefore propagates forward with a velocity equal to ξD. On the other hand, the propagation velocity according to Riemann is equal to $v+c$ and

$$v=v_1-\int_{p}^{p_1}\frac{dp}{\varrho c}.$$

Hence

$$\xi D=v+c.$$

Applying this immediately behind the detonation front we see that

1. The pressure p is either constant and equal to p_1 in a certain region immediately behind the detonation front, i. e. no rarefaction occurs immediately behind the front.

2. Or $D=v+c$ immediately behind the detonation front.

If a rarefaction occurrs immediately behind the front the Chapman-Jouguet condition must thus be fulfilled.

If the Chapman-Jouguet condition does not apply the detonation propagates with a velocity $D>D^*$. The section with constant pressure $p=p_1$ immediately behind the front extends from the front $\xi=1$ to ξ_f determined through the equation

$$v_1+c_1=\xi_f D;\ w/D\leqslant\xi_f<1. \tag{2}$$

We have to distinguish between two cases as the detonation gas immediately behind the detonation front is represented by a point above or below the Chapman-Jouguet point C (see the fig.).

If the gas is represented by the point A above C

$$v_1+c_1>D$$

applies, and the equation (2) cannot be satisfied. Thus $v+c=v_1+c_1$ and $p=p_1$ etc. will be valid in the entire region behind the front. This case can be realized by a suitable selection of the piston velocity w. As long as $w<v_1^*$ where v_1^* indicates the particle velocity immediately behind the front according to

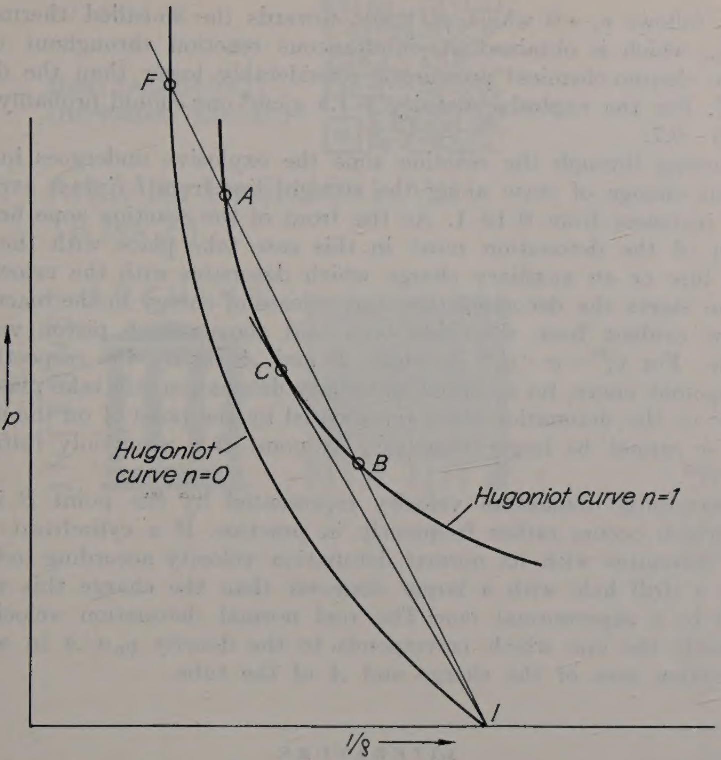

Fig. 1.

Chapman-Jouguet, the detonation velocity D remains equal to D^*. However, as soon as $w > v_1^*$, D becomes $> D^*$ and a constant state will prevail between the front and the piston ($p = p_1$). According to Döring [2], v. Neumann [3] and Zeldovitch [4] the front of the reaction zone will be situated at the point F on the Hugoniot curve $n = 0$ and the reaction must be assumed to start through the shock wave in the same way as for an ordinary detonation according to Chapman-Jouguet.

A driving piston of the kind in question here can for example be achieved by detonating a primary charge in contact with a secondary charge.

If, on the other hand, the state immediately behind the front is represented by the point B on the Hugoniot curve then

$$c_1 + v_1 < D$$

and equation (2) can consequently be fulfilled for a certain ξ_f. In the section $\xi_f \leqslant \xi \leqslant 1$ a constant state $p = p_1$ etc. will prevail. Behind ξ_f p and v will decrease and this implies that $v_1 > w$. With an increasing D p_1 will decrease and consequently also v_1 which is seen from the formula

$$\frac{p_1 - p_0}{\varrho_0} = v_1 D.$$

For $D \to \infty$ follows $v_1 \to 0$ whilst p_1 tends towards the so-called thermochemical pressure p_∞, which is obtained at simultaneous reaction throughout the entire charge. The thermo-chemical pressure is considerably lower than the detonation pressure p_1^*. For the explosive density 1–1.5 g/cm³ one should probably allow for $p_\infty / p_1^* = 0.6 - 0.7$.

When passing through the reaction zone the explosive undergoes in this case a continuous change of state along the straight line from I (intact explosive) to B when n increases from 0 to 1. As the front of the reaction zone lies in I the propagation of the detonation must in this case take place with the aid of a detonating fuse or an auxiliary charge which detonates with the velocity D and all the time starts the decomposition and release of energy in the reaction front.

It is now evident from what has been said above which piston velocities w are possible. For $v_1^{(B)} < w < v_1^{(A)}$ in which B and A signify the respective points on the Hugoniot curve, no so-called stationary detonation will take place. $w = v_1^{(A)}$ corresponds to the detonation state represented by the point A on the detonation curve and w cannot be larger than $v_1^{(A)}$. At point B w need only fulfil the condition $w \leqslant v_1^{(B)}$.

The supernormal detonation velocity represented by the point B is realized in a case which occurs rather frequently in practice. If a cylindrical charge of density ϱ_0 detonates with its normal detonation velocity according to Chapman-Jouguet in a drill hole with a larger diameter than the charge this velocity in reality will be a supernormal one. The real normal detonation velocity in the hole is namely the one which corresponds to the density $\varrho_0 a/A$ in which a is the cross-section area of the charge and A of the tube.

LITERATURE

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